

More Topics that may be on final of Probability

1. Problems like problems 4.7, 4.8, 4.9 but be able to graph the probability mass function.

2. Problem 4.10 where they are asking you to find $P(X=i|X>0)$. Note that they give you the probability mass function when they give you for instance $P(-2) = 1/11$. This statement really means that $P(\{X=-2\}) = 1/11$. Note that there are many ways to express the probability mass function and you have to be fluent in all of them. In this problem graph the probability mass function. What is the cumulative distribution function (what we have been calling F or F_X vs f_X).

3. Problem like 4.25 except find the $\text{Var}(X)$ too. Also you should be able to do this problem if it is expressed in a different way. Let $X_1=1$ if you get heads on first flip and 0 otherwise. Note that they are saying $P(\{X_1=1\}) = .6$. Let $X_2=1$ if you get a head on the second flip. The problem is saying $P(\{X_2=1\}) = .7$. $X = X_1 + X_2$. Use our theorems to find $E(X_1 + X_2)$ and $\text{Var}(X_1 + X_2)$. If $A = (X_1 + X_2)/2$ what is the $E(A)$, $\text{Var}(A)$? What would happen if you had another independent Bernoulli flip (3 flips) where $P(\{X_3=1\}) = .4$? Find the expectation and variance of the number of heads and the average number of heads in this case. Do it by brute force where you first show me the probability mass function and use definitions of Expectation and Variance and then use our theorems for sums and variances.

4. Be able to explain why Chebyshev's theorem tells us that the standard deviation is an indication of the spread of a function.

5. Be able to prove that $E(X_1 + X_2) = E(X_1) + E(X_2)$. What makes this a little tricky to prove this using the theorem that $E(X) = \sum_i i \cdot P(\{X=i\})$ (using probability mass function) rather than the basic definition that $E(X) = \sum_{\omega \in \Omega} X(\omega) P(\omega)$. Relate this when X_i are grades of students in Test 1 and Test 3. Grad students should be able to demonstrate to me how to prove this theorem using the probability mass function.

6. Understand and be able to prove that $E(cX) = c \cdot E(X)$, $E(X+a) = E(X) + a$ and $\text{Var}(cX) = c^2 \cdot \text{Var}(X)$. Show that some criterion is necessary to have $\text{Var}(X_1 + X_2) = \text{Var}(X_1) + \text{Var}(X_2)$? What criterion do we use?

7. Understand why our theorems imply that when you have a sum of independent identically distributed random variables (think independent flips of a coin) that the variance of the Average of these variables approaches zero as you average more and more of the variables. This is one of the basis of descriptive statistics.